**PERIODIC MOTION**

Vibratory motion or vibration is a type of motion where the particle undergoes to and fro motion. It could be regular vibration which repeats itself over and over during a given length of time (like the motion of a simple pendulum or the piston of a gasoline engine) or irregular vibration (like the shaking of a building when there is an earthquake).

CHARACTERISTICS OF A BODY UNDERGOING PERIODIC MOTION:

1. Equilibrium position – that position of the body when the forces acting on it have zero resultant. This is actually the position the body takes when it is at rest.
2. Restoring force, *F* – the force that brings back (restores) the body to its equilibrium position. It is always directed towards the equilibrium position.
3. Displacement, *x or y* – the distance of the body from the equilibrium position at any instant.
4. Amplitude, *A* – the maximum displacement from the equilibrium
5. One cycle of motion or one vibration – that part of the motion that is repeated
6. Period, *T –* the time it takes for a displaced object to make a complete oscillation back and forth about its equilibrium position. (sec)
7. Frequency, *f* – the reciprocal of period, the number of oscillations per second (cycles/sec or vib/sec or Hertz(Hz))

1. Angular frequency (or speed), *ω* = rate of change of an angular quantity that is always measured in radians

ω = 2πf rad/s

When we are confronted by a condition wherein the restoring force is directly proportional to the displacement then we have SIMPLE HARMONIC MOTION.

**SIMPLE HARMONIC MOTION**

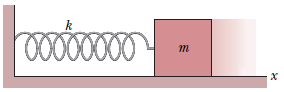
* Simplest type of periodic motion
* Restoring force is directly proportional to displacement
* Velocity of the body is inversely proportional to displacement
* Acceleration of the body is directly proportional to the negative of the displacement
* *To help us visualize the concepts, we will be using as models body attached to a spring and the simple pendulum.*

**EQUATIONS FOR SHM:**

1. **MASS ATTACHED TO A SPRING**

* If a body attached to a spring is displaced from its equilibrium position, the spring exerts a *restoring force* on it, which tends to restore the object to the equilibrium position.

Mathematically, the restoring force **F** is given by

where k is the spring constant ([N](http://en.wikipedia.org/wiki/Newton_(unit))/m), and **x** is the displacement from the equilibrium position (in m).

At equilibrium position, the mass has momentum because of the impulse that the restoring force has imparted. Therefore, the mass continues past the equilibrium position, compressing the spring. A net restoring force then tends to slow it down, until its velocity vanishes, whereby it will attempt to reach equilibrium position again. As long as the system has no energy loss, the mass will continue to oscillate.

***Energy in Simple Harmonic Motion***

By Law of Conservation of Energy: the total energy of a vibrating body is constant.

Total Energy = K + U

K = ½ mv2 Joules

where: K = kinetic energy; m = mass of the body

v = velocity of the body

U = ½ kx2 Joules

U = potential energy; k = spring constant

x = displacement of the body

*Consider the body at the extreme position or at its maximum displacement (x = A):*

Total Energy = U = ½ kA2

v = 0:

Because Total Energy is Constant:

**½ kA2 = ½ mv2 + ½ kx2**

The *instantaneous* ***VELOCITY***of the vibrating bodyis determined via the above energy equation as:

**v =**

**v =**

but = ω

The *instantaneous* ***ACCELERATION***is determined via Hooke’s Law, F = -kx, and F = ma;

Equating these two expressions for F gives:

**a = - = -ω2x**

From the equations of angular frequency, *ω = 2πf* and *ω =*  , other equation to solve for frequency and period are as follows:

**Frequency Period**

**T =**

**f =**

**Displacement, Velocity and Acceleration as a Function of Time, t**

1. Displacement:

**x(t) = Acos(ωt *± Ø*)**

**A =**

from the energy equation: where: x0 = initial displacement

v0 = initial speed

**v(t) = - ωAsin(ωt *± Ø*)**

1. Velocity: v =
2. Acceleration: a = =

**a(t) = - ω2Acos(ωt *± Ø*)**

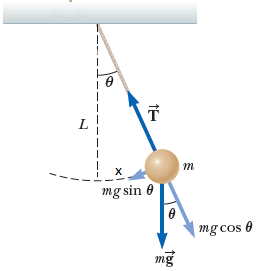
Solving for the Phase Angle, ***Ø*** at an initial condition, t = 0:

=

***Ø* = tan-1**

1. **SIMPLE PENDULUM**

* Consists of a point mass suspended by a weightless, unstretchable string in a uniform gravitational field whose path is not a straight line but an arc.
* If the pendulum swings with a small angle with the vertical, its motion is simple harmonic.



The restoring force is the net force on the bob, equal to the component of the weight, mg, tangent to the arc:

* Small angle approximation:
* The displacement of the pendulum along the arc is given by

This equation fits Hooke's law, F = -kx. The effective force constant is

* Angular frequency

* Frequency

**f =**

* Period

**T = 2π**

Where: = length of the string

g = acceleration due to gravity

* Laws of the simple pendulum (for small displacements)

1. The angle of swing (amplitude) does not affect the period.
2. The mass of the body does not affect the period.
3. The period is directly proportional to the square root of its length.
4. The period is inversely proportional to the square root of the acceleration.